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Mixed convection boundary-layer flow from a horizontal circular cylinder in micropolar fluids: case of constant wall temperature

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Abstract *The laminar mixed convection boundary-layer flow of a micropolar fluid past a horizontal circular cylinder in a stream flowing vertically upwards has been studied in both cases of a heated and cooled cylinder. The solutions for the flow and heat transfer characteristics are evaluated numerically for different parameters, such as the mixed convection parameter* λ*, the material parameter* K *(vortex viscosity parameter) and the Prandtl number* $Pr = 1$ *and 6.8, respectively. It is found, as for the case of a Newtonian fluid considered for* $Pr = 1$*, that heating the cylinder delays separation and can, if the cylinder is warm enough, suppress it completely. Cooling the cylinder, on the other side, brings the separation point nearer to the lower stagnation point and for sufficiently cold cylinder there will not be a boundary-layer on the cylinder. This model problem may solve industrial problems with processing of polymeric liquids, lubricants and molten plastics.*

Nomenclature

a = radius of the cylinder
C_f = local skin friction coefficient
g = acceleration due to gravity
Gr = Grashof number
H = non-dimensional microrotation component normal to the (x, y)-plane
j = microinertia density
K = material parameter
n = constant
Pr = Prandtl number
Re = Reynolds number
Q_w = heat transfer coefficient
T = fluid temperature
u, v = non-dimensional velocity components along x and y directions, respectively
$u_e(x)$ = non-dimensional velocity outside boundary-layer

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U_∞ = free stream velocity
x, y = non-dimensional Cartesian coordinates along the surface of the cylinder and normal to it, respectively

Greek symbols
α = thermal diffusivity
β = thermal expansion coefficient
γ = spin gradient viscosity
λ = mixed convection parameter
θ = non-dimensional temperature
κ = vortex viscosity
μ = dynamic viscosity
ν = kinematic viscosity
ρ = density
ψ = non-dimensional stream function

Superscripts
$'$ = differentiation with respect to y
$\bar{}$ = dimensional variables

Subscripts
w = condition at the wall
∞ = ambient condition

Introduction

It is well known that the classical Navier-Stokes theory does not adequately describe the flow properties of polymeric fluids, i.e. fluids containing certain additives and some naturally occurring fluids, such as animal blood. Such fluids, which exhibit certain microscopic effects arising from the local structure and microrotations of fluid elements, are known as micropolar or thermomicropolar fluids. They exhibit the microrotational effects and microrotational inertia and can support couple stresses and body couples only. Micropolar fluid theory and its extension to thermomicropolar fluids was formulated by Eringen (1966, 1972, 2001) and has received considerable interest in the recent years due to its many practical applications. It can be used to study the behaviour of exotic lubricants, colloidal suspensions or polymeric additives, blood flow, liquid crystals and dirty oils, to name just a few practical applications of this theory. The theory of micropolar fluids has generated a lot of interests and many flows problems have been studied [see the review paper by Ariman *et al.* (1973), the book by Łukaszewicz (1999) and the recent papers by Nazar *et al.* (2001a, b)]. However, studies of the external convective flows of micropolar fluids have focused mainly on either pure forced or pure free convection problems (Ahmadi, 1976; Gorla *et al.*, 1998; Hossain *et al.*, 1999, 2000; Mathur *et al.*, 1978; Pop *et al.*, 1998; Rees and Pop, 1998). Relatively few studies have considered problems of combined forced and free convection (mixed convection) in micropolar fluids (Arafa and Gorla, 1992; Gorla, 1988, 1992; Hossain and Chowdhury, 1998; Wang, 1993).

Mixed convection flow from horizontal cylinders constitutes an important heat transfer problem from the standpoint of engineering applications and numerical analyses. An immense body of literature exists for the case of Newtonian fluids (Gebhart *et al.*, 1988; Pop and Ingham, 2001). However, relatively fewer investigations of mixed convection have been conducted, especially for the case of micropolar fluids. Recently, Mansour and Gorla (1999) have considered the problem of MHD mixed convection flow near the lower stagnation point of a horizontal circular cylinder immersed in a micropolar fluid using the similarity equations.

Therefore, the purpose of this paper is to present a theoretical study of the flow of a micropolar fluid over an impermeable horizontal circular cylinder, which is held at a constant temperature T_w surrounded by fluid at temperature T_∞, with the uniform free stream flowing in the upward vertical direction. The formulation of the problem follows that of Merkin (1977) for a similar problem of a Newtonian fluid past a horizontal cylinder. For large Reynolds and Grashof numbers, the equations governing the flow are the boundary-layer equations. These equations are solved numerically using the same method as that recently used by the present authors (Nazar *et al.*, 2001a, b) when studying the free convection boundary-layer flow past a horizontal circular cylinder in a micropolar fluid, namely, the Keller-box scheme.

It is shown that the solution depends on three parameters, namely, the mixed convection parameter, $|\lambda| = \mathrm{Gr}/\mathrm{Re}^2$, with $\lambda > 0$ ($T_w > T_\infty$) for a heated cylinder and $\lambda < 0$ ($T_w < T_\infty$) for a cooled cylinder, respectively, the material parameter, K, and the Prandtl number, Pr. For small values of $|\lambda|$ forced convection effects dominate, while for large values of $|\lambda|$ it is the natural convection, which is important, so that values of $|\lambda|$ of O(1), where both effects are comparable, are of most interest. We present the principal results in the form of tables for the heat transfer and skin friction coefficients, while the results for the position of boundary-layer separation are presented in figures. It is shown that for a heated cylinder ($\lambda > 0$) the separation of the boundary-layer is delayed for each value of K considered, and it is found that there is a value of $\lambda = \lambda_K$ for which the boundary-layer does not separate at all. On the other hand, for a cooled cylinder ($\lambda < 0$) the buoyancy forces retard the fluid and therefore the position of the boundary-layer separation is brought nearer to the lower stagnation point of the cylinder. A unique value of $\lambda = \lambda_0$ is found for each given value of K for which the boundary-layer separates at this point. For values of λ less than λ_0 a boundary-layer solution is not possible. The results are given for $\mathrm{Pr} = 1$ and $\mathrm{Pr} = 6.8$, respectively. We mention that results for $\mathrm{Pr} = 1$ are given in order that we can compare the present results for $K = 0$ (Newtonian fluid) with those of Merkin (1977), and we have shown that our results are in excellent agreement with those of Merkin (1977). It is also worth mentioning that the value of $\mathrm{Pr} = 6.8$ for a Newtonian fluid ($K = 0$) corresponds to water at 21°C (Holman, 1997).

Basic equations

Consider a horizontal circular cylinder of radius a, which is heated to a constant temperature T_w and is immersed in a viscous and incompressible micropolar fluid of uniform free stream U_∞ and ambient temperature T_∞, as shown in Figure 1. The orthogonal coordinates x and y are measured along the surface of the cylinder, starting with the lower stagnation point, and normal to it, respectively. It is assumed that the Boussinesq and boundary layer approximations are valid. Under these assumptions, the equations governing the steady mixed convection boundary-layer flow are

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0 \tag{1}$$

$$\rho\left(\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}}\right) = \rho \overline{u}_e \frac{d\overline{u}_e}{d\overline{x}} + (\mu + \kappa)\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + \rho g \beta (T - T_\infty)\sin\left(\frac{\overline{x}}{a}\right) + \kappa \frac{\partial \overline{H}}{\partial \overline{y}} \tag{2}$$

$$\overline{u}\frac{\partial T}{\partial \overline{x}} + \overline{v}\frac{\partial T}{\partial \overline{y}} = \alpha \frac{\partial^2 T}{\partial \overline{y}^2} \tag{3}$$

$$\rho j\left(\overline{u}\frac{\partial \overline{H}}{\partial \overline{x}} + v\frac{\partial \overline{H}}{\partial \overline{y}}\right) = -\kappa\left(2\overline{H} + \frac{\partial \overline{u}}{\partial \overline{y}}\right) + \gamma \frac{\partial^2 \overline{H}}{\partial \overline{y}^2} \tag{4}$$

subject to the boundary conditions

$$\overline{u} = \overline{v} = 0, \quad T = T_{\mathrm{w}}, \quad \overline{H} = -n\frac{\partial \overline{u}}{\partial \overline{y}} \quad \text{on } \overline{y} = 0 \tag{5a}$$

$$\overline{u} \to \overline{u}_e(\overline{x}), \quad T \to T_\infty, \quad \overline{H} \to 0 \quad \text{as } \overline{y} \to \infty \tag{5b}$$

where $(\overline{u}, \overline{v})$ are the velocity components along the $(\overline{x}, \overline{y})$ axes, $\overline{u}_e(\overline{x})$ is the velocity outside the boundary-layer, and T, ρ, g, β, μ, κ, $\overline{H}$, j and α are the fluid temperature, density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity, microrotation component of the micropolar fluid normal to $(\overline{x}, \overline{y})-$plane, microinertia density and thermal diffusivity of the fluid, respectively. It is worth mentioning that in equation 5(a) we have followed Arafa and Gorla (1992) by assigning a variable relation

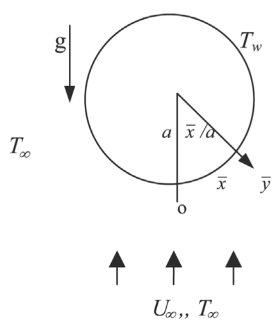

Figure 1.
Physical model and coordinate system

between microrotation and the surface skin friction, where n is a constant and $0 \leq n \leq 1$. The value $n = 0$, which indicates $\overline{H}(\overline{x}, 0) = 0$, represents concentrated particle flows in which the particle density is sufficiently great that microelements close to the wall are unable to rotate (Jena and Mathur, 1981). This condition is also called "strong" interaction (Guram and Smith, 1980). The case corresponding to $n = {}^1/_2$ results in the vanishing of antisymmetric part of the stress tensor and represents weak concentration (Ahmadi, 1976). In this case, the particle rotation is equal to fluid vorticity at the boundary for fine particle suspension. When $n = 1$, we have flows which are representative of turbulent boundary layers (Peddieson, 1972). The case of $n = {}^1/_2$ is considered in the present study due to the paper limitation.

It is implied that the microinertia j is constant and that the microstructure spin effect is included in $\overline{H}$. We assume that the coefficients of viscosity are related as (Ahmadi, 1976)

$$\gamma = (\mu + \kappa/2)j \tag{6}$$

and that a uniform stream $\left({}^1/_2\right)U_\infty$ is flowing vertically upwards over the cylinder, so that the free stream velocity $\overline{u}_e(\overline{x})$ for the boundary-layer equations (Merkin, 1977) is

$$\overline{u}_e(\overline{x}) = U_\infty \sin(\overline{x}/a) \tag{7}$$

Equations (1)-(4) are now non-dimensionalised using the variables

$$x = \overline{x}/a, \quad y = \mathrm{Re}^{1/2}(\overline{y}/a), \quad u = \overline{u}/U_\infty, \quad v = \mathrm{Re}^{1/2}\overline{v}/U_\infty \tag{8a}$$

$$H = \mathrm{Re}^{-1/2}(a/U_\infty)\overline{H}, \quad \theta = (T - T_\infty)/(T_\mathrm{w} - T_\infty), \quad u_e(x) = \overline{u}_e(\overline{x})/U_\infty \tag{8b}$$

where $\mathrm{Re} = U_\infty a/v$ is the Reynolds number. The microinertia density j is taken to be $j = av/U_\infty$. Substitution of equation (8) into equations (1)-(4) leads to the following non-dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\mathrm{d}u_e}{\mathrm{d}x} + (1 + K)\frac{\partial^2 u}{\partial y^2} + \lambda\theta\sin x + K\frac{\partial H}{\partial y} \tag{10}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

$$u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}=-K\left(2H+\frac{\partial u}{\partial y}\right)+\left(1+\frac{K}{2}\right)\frac{\partial^2 H}{\partial y^2} \tag{12}$$

where λ is the mixed convection parameter and K is termed as the vortex viscosity or material parameter, which are defined as

$$\lambda=\mathrm{Gr}/\mathrm{Re}^2, \quad K=\kappa/\mu \tag{13}$$

with $\mathrm{Gr}=g\beta(T_{\mathrm{w}}-T_\infty)\,a^3/v^2$ being the Grashof number; $\lambda>0$ ($T_{\mathrm{w}}>T_\infty$) for a heated cylinder and $\lambda<0$ ($T_{\mathrm{w}}<T_\infty$) for a cooled cylinder, respectively. The boundary conditions (equation (5)) become

$$u=v=0, \quad \theta=1, \quad H=-\frac{1}{2}\frac{\partial u}{\partial y} \quad \text{on } y=0 \tag{14a}$$

$$u\to u_e(x), \quad \theta\to 0, \quad H\to 0 \quad \text{as } y\to\infty \tag{14b}$$

We notice at this place that for $K=0$, where the flow and temperature fields are unaffected by the microstructure of the fluid, and the microrotation component is a passive quantity, the present problem reduces to that of a viscous fluid and this case has been studied by Merkin (1977).

To solve equations (9)-(12), subject to the boundary conditions (equation (14)), we assume the following variables

$$\psi=xF(x,y), \quad \theta=\theta(x,y), \quad H=xG(x,y) \tag{15}$$

where ψ is the stream function defined in the usual way as

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x} \tag{16}$$

Using equations (10), (11), (12), (15) and (16), we get, after some algebra, the resulting equations

$$(1+K)\frac{\partial^3 F}{\partial y^3}+F\frac{\partial^2 F}{\partial y^2}-\left(\frac{\partial F}{\partial y}\right)^2+\frac{\sin x\cos x}{x}+\lambda\frac{\sin x}{x}\theta+K\frac{\partial G}{\partial y}$$
$$=x\left(\frac{\partial F}{\partial y}\frac{\partial^2 F}{\partial x\,\partial y}-\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}\right) \tag{17}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+F\frac{\partial\theta}{\partial y}=x\left(\frac{\partial F}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial F}{\partial x}\frac{\partial\theta}{\partial y}\right) \tag{18}$$

$$\left(1+\frac{K}{2}\right)\frac{\partial^2 G}{\partial y^2}+F\frac{\partial G}{\partial y}-\frac{\partial F}{\partial y}G-K\left(2G+\frac{\partial^2 F}{\partial y^2}\right)$$
$$=x\left(\frac{\partial F}{\partial y}\frac{\partial G}{\partial x}-\frac{\partial F}{\partial x}\frac{\partial G}{\partial y}\right) \tag{19}$$

subject to the boundary conditions

$$F=\frac{\partial F}{\partial y}=0, \quad \theta=1, \quad G=-\frac{1}{2}\frac{\partial^2 F}{\partial y^2} \quad \text{on } y=0 \tag{20a}$$

$$\frac{\partial F}{\partial y}\to\frac{\sin x}{x}, \quad \theta\to 0, \quad G\to 0 \quad \text{as } y\to\infty \tag{20b}$$

It can be seen that near the lower stagnation point of the cylinder, i.e. $x \approx 0$, equations (17)-(19) reduce to the following ordinary differential equations:

$$(1+K)F''' + FF'' - F'^2 + 1 + \lambda\theta + KG' = 0 \tag{21}$$

$$\frac{1}{\Pr}\theta'' + F\theta' = 0 \tag{22}$$

$$\left(1+\frac{K}{2}\right)G'' + FG' - F'G - K(2G + F'') = 0 \tag{23}$$

subject to the boundary conditions

$$F(0)=F'(0)=0, \quad \theta(0)=1, \quad G(0)=-\tfrac{1}{2}F''(0) \tag{24a}$$

$$F'\to 1, \quad \theta\to 0, \quad G\to 0 \quad \text{as } y\to\infty \tag{24b}$$

where primes denote differentiation with respect to y.

In practical applications, the physical quantities of principal interest are the heat transfer and skin friction coefficients, which are defined, in non-dimensional form, as

$$Q_w = \mathrm{Re}^{-1/2}\frac{aq_w}{k(T_w - T_\infty)}, \quad C_f = \mathrm{Re}^{1/2}\frac{\tau_w}{\rho U_\infty^2} \tag{25}$$

where k is the thermal conductivity of the fluid, and q_{w} and τ_{w} are the heat flux from the surface of the cylinder and skin friction, respectively, given by

$$q_{\mathrm{w}} = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad \tau_{\mathrm{w}} = \left[(\mu+\kappa)\frac{\partial \bar{u}}{\partial \bar{y}} + \kappa \bar{H}\right]_{\bar{y}=0} \tag{26}$$

Using the non-dimensional variables (equation (8)) and the transformation (equation (15)), we get

$$Q_{\mathrm{w}} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}, \quad C_{\mathrm{f}} = \left(1+\frac{K}{2}\right)x\left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0} \tag{27}$$

For large values of λ ($\gg 1$), a solution of equations (21)-(23) can be found using the transformation

$$F(y) = \lambda^{1/4} f(\eta), \quad \theta(y) = g(\eta), \quad H(y) = \lambda^{3/4} h(\eta), \quad \eta = \lambda^{1/4} y \tag{28}$$

Substituting equation (28) into equations (21)-(23) they become

$$(1+K)f''' + ff'' - f'2 + \lambda^{-1} + \theta + Kh' = 0 \tag{29}$$

$$\frac{1}{\mathrm{Pr}} g'' + fg' = 0 \tag{30}$$

$$\left(1+\frac{K}{2}\right)h'' + fh' - f'h - K\lambda^{-1/2}(2h + f'') = 0 \tag{31}$$

subject to the boundary conditions

$$f(0) = f'(0) = 0, \quad g(0) = 1, \quad h(0) = -\tfrac{1}{2} f''(0) \tag{32a}$$

$$f' \to \lambda^{-1/2}, \quad g \to 0, \quad h \to 0 \quad \text{as} \quad \eta \to \infty \tag{32b}$$

where primes now denote differentiation with respect to η. A solution of equations (29)-(31) is sought in the form of series

$$\begin{aligned} f &= f_0(\eta) + \lambda^{-1/2} f_1(\eta) + \lambda^{-1} f_2(\eta) + \ldots \\ g &= g_0(\eta) + \lambda^{-1/2} g_1(\eta) + \lambda^{-1} g_2(\eta) + \ldots \\ h &= h_0(\eta) + \lambda^{-1/2} h_1(\eta) + \lambda^{-1} h_2(\eta) + \ldots \end{aligned} \tag{33}$$

for $\lambda \gg 1$. The coefficient functions f_i, g_i and h_i ($i = 0, 1, 2$) are given by three sets of ordinary differential equations with the corresponding boundary conditions resulting from equation (32). Solving these sets of equations, we get

$$F''(0) = \lambda^{3/4}[f''_0(0) + \lambda^{-1/2} f''_1(0) + \lambda^{-1} f''_2(0) + \ldots]$$
$$\theta'(0) = \lambda^{1/4}[g'_0(0) + \lambda^{-1/2} g'_1(0) + \lambda^{-1} g'_2(0) + \ldots] \quad (34)$$

for $\lambda \gg 1$.

Results and discussion

Equations (17)-(19) subject to the boundary conditions (equation (20)) have been solved numerically using a very efficient implicit finite-difference method known as the Keller-box scheme along with the Newton's linearization technique as described by Cebeci and Bradshaw (1984). The numerical solutions start at the lower stagnation point of the cylinder, $x \approx 0$, with initial profiles as given by equations (21)-(23) along with the boundary conditions (equation (24)) and proceed round the cylinder up to the separation point. Representative results for the wall temperature coefficient, Q_w, and the skin friction coefficient, C_f, have been obtained for the following values of the vortex viscosity or material parameter $K = 0$ (Newtonian fluid), 0.5, 1.0 and 2.0 at different positions x with various values of the mixed convection parameter λ when $Pr = 1.0$ and 6.8, respectively. We notice that the values of the parameter K satisfy the thermodynamic restriction noted by Eringen (1972).

In order to verify the accuracy of the present method, the values of Q_w, C_f, $F''(0)$ and $-\theta'(0)$, as given by expressions (27) and (34), are compared in Tables I, II and V with those reported by Merkin (1977) for $K = 0$ (Newtonian fluid), $Pr = 1$ and various values of x and λ. It is seen that the results are found to be in excellent agreement. The variation of Q_w and C_f with x is also illustrated for $K = 0$ and $Pr = 1$ in Figures 2 and 3, and these figures show again an excellent agreement between the present results and those obtained by Merkin (1977). We are, therefore, confident that the present results are very accurate. Also the values of Q_w, C_f, $F''(0)$ and $-\theta'(0)$ for $K = 0$ and $Pr = 6.8$ (water at 21°C) are shown in Tables III-V, respectively. We notice that for $K = 0$ (Newtonian fluid), the values of the heat transfer coefficient Q_w are higher for $Pr = 6.8$ than for $Pr = 1.0$, while the values of the skin friction coefficient C_f are lower when $\lambda > 0$ and higher when $\lambda < 0$ for $Pr = 6.8$ than for $Pr = 1.0$.

Further values of Q_w and C_f for $K = 0.5$ and 2 (micropolar fluids) at different positions x and various values of the parameter λ can be found from Tables VI-XIII when the Prandtl number $Pr = 1$ and 6.8, respectively. It can be seen from these tables that the values of Q_w are lower, while those of C_f are higher for micropolar fluids than those for a Newtonian fluid ($K = 0$) when the parameters x and λ are fixed. We can also conclude from these tables that, as it is expected, the boundary-layer separates from cylinder for some negative values of λ (cooling cylinder) and also for some positive values of λ (heated

x	λ −1.75	−1.5	−1.0	−0.5	0.0	0.5	0.88	0.89	1.0	2.0	5.0
0.0	0.4205	0.4601	0.5080	0.5430	0.5710	0.5949	0.6112	0.6116	0.6160	0.6518	0.7320
	(0.4199)	(0.4576)	(0.5067)	(0.5420)	(0.5705)	(0.5943)	(0.6096)	(0.6100)	(0.6156)	(0.6497)	(0.7315)
0.2	0.4035	0.4510	0.5022	0.5382	0.5668	0.5912	0.6076	0.6080	0.6125	0.6487	0.7292
	(0.4059)	(0.4498)	(0.5018)	(0.5380)	(0.5668)	(0.5911)	(0.6067)	(0.6071)	(0.6115)	(0.6471)	(0.7261)
0.4		0.4235	0.4862	0.5257	0.5560	0.5811	0.5980	0.5984	0.6031	0.6402	0.7218
		(0.4236)	(0.4865)	(0.5260)	(0.5564)	(0.5817)	(0.5979)	(0.5983)	(0.6028)	(0.6393)	(0.7193)
0.6		0.3239	0.4584	0.5047	0.5380	0.5649	0.5826	0.5831	0.5880	0.6265	0.7100
		(0.3373)	(0.4594)	(0.5056)	(0.5391)	(0.5661)	(0.5833)	(0.5837)	(0.5885)	(0.6264)	(0.7082)
0.8			0.4140	0.4747	0.5130	0.5425	0.5615	0.5620	0.5673	0.6081	0.6939
			(0.4160)	(0.4760)	(0.5145)	(0.5443)	(0.5631)	(0.5636)	(0.5686)	(0.6086)	(0.6929)
1.0			0.3259	0.4338	0.4808	0.5142	0.5350	0.5355	0.5414	0.5850	0.6739
			(0.3326)	(0.4353)	(0.4826)	(0.5165)	(0.5375)	(0.5380)	(0.5435)	(0.5863)	(0.6737)
1.2				0.3765	0.4406	0.4800	0.5033	0.5040	0.5105	0.5578	0.6505
				(0.3784)	(0.4426)	(0.4828)	(0.5066)	(0.5072)	(0.5133)	(0.5597)	(0.6509)
1.4				0.2670	0.3909	0.4400	0.4668	0.4675	0.4750	0.5270	0.6238
				(0.2736)	(0.3928)	(0.4431)	(0.4709)	(0.4716)	(0.4785)	(0.5294)	(0.6248)
1.6					0.3262	0.3940	0.4260	0.4267	0.4354	0.4931	0.5943
					(0.3280)	(0.3972)	(0.4307)	(0.4314)	(0.4394)	(0.4960)	(0.5959)
1.8					0.2049	0.3410	0.3810	0.3820	0.3924	0.4570	0.5624
					(0.2114)	(0.3444)	(0.3863)	(0.3872)	(0.3967)	(0.4601)	(0.5645)
2.0						0.2787	0.3326	0.3338	0.3465	0.4195	0.5288
						(0.2821)	(0.3383)	(0.3394)	(0.3509)	(0.4225)	(0.5311)
2.2						0.1920	0.2812	0.2827	0.3002	0.3814	0.4935
						(0.1970)	(0.2871)	(0.2885)	(0.3029)	(0.3842)	(0.4959)
2.4							0.2273	0.2300	0.2515	0.3438	0.4567
							(0.2331)	(0.2350)	(0.2540)	(0.3460)	(0.4592)
2.6							0.1711	0.1750	0.2040	0.3074	0.4181
							(0.1766)	(0.1796)	(0.2061)	(0.3088)	(0.4205)
2.8							0.1115	0.1196	0.1636	0.2726	0.3767
							(0.1162)	(0.1227)	(0.1634)	(0.2730)	(0.3790)
3.0								0.0824	0.1397	0.2386	0.3297
								(0.0745)	(0.1354)	(0.2381)	(0.3321)
π								0.1090	0.1380	0.2135	0.2860
								(0.1033)	(0.1306)	(0.2122)	(0.2918)

Table I. Values of heat transfer coefficient Q_w for $K=0$ (Newtonian fluid), $\Pr = 1$ and various values of λ [results in parenthesis are those of Merkin (1977)]

cylinder). Cooling the cylinder brings the separation point close to the lower stagnation point and for sufficiently large negative values of λ or sufficiently cold cylinder, there will not be a boundary layer on the cylinder. Increasing λ delays the separation and that separation can be suppressed completely in $0 < x < \pi$ for sufficiently large values of λ (>0). The numerical values of the reduced skin friction $F''(0)$ and reduced heat transfer $-\theta'(0)$ near the lower stagnation point of the cylinder obtained by solving equations (21)-(23) for the same values of K and Pr, and some values of λ are given in Tables XIV and XV. The values obtained from the asymptotic series (equation (34)) for λ ($\gg 1$) are

x	λ −1.75	−1.5	−1.0	−0.5	0.0	0.5	0.88	0.89	1.0	2.0	5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
0.2	0.0053	0.0531	0.1252	0.1865	0.2421	0.2940	0.3313	0.3323	0.3430	0.4347	0.6794
	(0.0066)	(0.0533)	(0.1257)	(0.1871)	(0.2427)	(0.2945)	(0.3321)	(0.3330)	(0.3436)	(0.4354)	(0.6803)
0.4		0.0720	0.2242	0.3486	0.4602	0.5635	0.6381	0.6400	0.6610	0.8434	1.3282
		(0.0741)	(0.2266)	(0.3511)	(0.4627)	(0.5662)	(0.6409)	(0.6429)	(0.6639)	(0.8464)	(1.3318)
0.6		0.0002	0.2731	0.4653	0.6337	0.7883	0.8992	0.9022	0.9335	1.2040	1.9200
		(0.0026)	(0.2784)	(0.4706)	(0.6393)	(0.7941)	(0.9057)	(0.9085)	(0.9398)	(1.2106)	(1.9277)
0.8			0.2463	0.5185	0.7461	0.9515	1.0980	1.1018	1.1432	1.4983	2.4316
			(0.2554)	(0.5271)	(0.7552)	(0.9614)	(1.1088)	(1.1125)	(1.1538)	(1.5094)	(2.4447)
1.0			0.0890	0.4935	0.7855	1.0420	1.2227	1.2275	1.2785	1.7135	2.8460
			(0.1069)	(0.5051)	(0.7982)	(1.0561)	(1.2383)	(1.2430)	(1.2938)	(1.7295)	(2.8648)
1.2				0.3745	0.7460	1.0545	1.2683	1.2740	1.3343	1.8430	3.1514
				(0.3890)	(0.7615)	(1.0727)	(1.2886)	(1.2941)	(1.3541)	(1.8637)	(3.1761)
1.4				0.1025	0.6256	0.9907	1.2365	1.2430	1.3121	1.8868	3.3430
				(0.1253)	(0.6429)	(1.0121)	(1.2608)	(1.2671)	(1.3356)	(1.9117)	(3.3729)
1.6					0.4229	0.8583	1.1353	1.1437	1.2200	1.8515	3.4215
					(0.4405)	(0.8814)	(1.1625)	(1.1695)	(1.2459)	(1.8793)	(3.4557)
1.8					0.0833	0.6696	0.9788	0.9870	1.0728	1.7488	3.3931
					(0.1069)	(0.6927)	(1.0072)	(1.0491)	(1.0986)	(1.7781)	(3.4300)
2.0						0.4385	0.7852	0.7941	0.8880	1.5945	3.2674
						(0.4599)	(0.8131)	(0.8295)	(0.9117)	(1.6236)	(3.3053)
2.2						0.1642	0.5757	0.5850	0.6866	1.4062	3.0555
						(0.1842)	(0.6012)	(0.6103)	(0.7063)	(1.4334)	(3.0928)
2.4							0.3722	0.3830	0.4900	1.2010	2.7682
							(0.3936)	(0.4033)	(0.5048)	(1.2248)	(2.8033)
2.6							0.1955	0.2074	0.3215	0.9927	2.4133
							(0.2112)	(0.2219)	(0.3287)	(1.0123)	(2.4447)
2.8							0.0624	0.0770	0.1949	0.7893	1.9923
							(0.0711)	(0.0847)	(0.1979)	(0.8043)	(2.0188)
3.0								0.0228	0.1355	0.5897	1.4944
								(0.0149)	(0.1292)	(0.6002)	(1.5154)
π								0.0610	0.1325	0.4404	1.0760
								(0.0504)	(0.1206)	(0.4508)	(1.0919)

Table II.
Values of skin friction coefficient C_f for $K=0$ (Newtonian fluid), Pr = 1 and various values of λ [results in parenthesis are those of Merkin (1977)]

also included in this table and these show good agreement with the numerical values even at moderate values of λ.

The variation of the separation point x_s with λ is plotted in Figures 4 and 5 for $K = 0$ (Newtonian fluid), 0.5, 1 and 2 with Pr = 1.0 and 6.8, respectively. The actual value of $\lambda = \lambda_K$ which first gives no separation is difficult to determine exactly as it has to be found by successive integrations of the equations. However, the numerical solutions indicate that the value of λ_K which first gives no separation lies between 0.88 and 0.89 for $K = 0.0$, between 0.90 and 0.91 for $K = 0.5$, and between 0.94 and 0.95 for $K = 2.0$ when Pr = 1.0

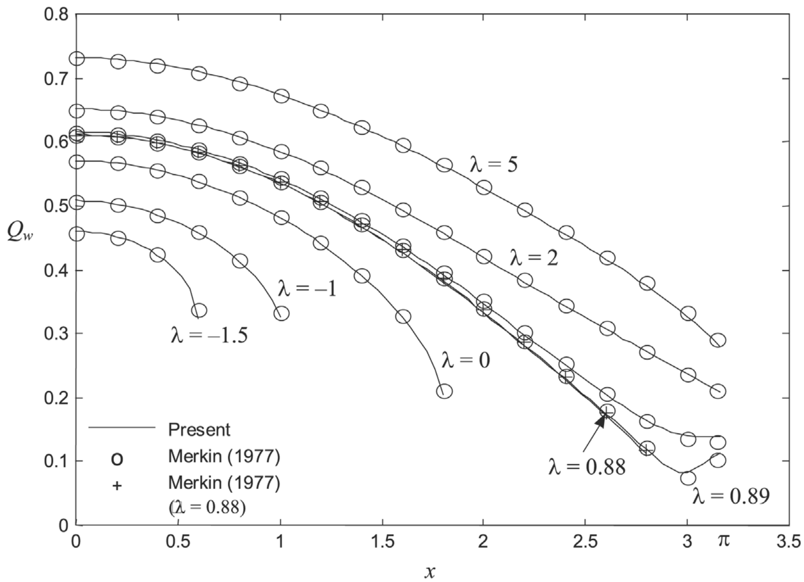

Figure 2. Comparison of the heat transfer coefficient Q_w for $K=0$ (Newtonian fluid), $Pr = 1$ and various values of λ

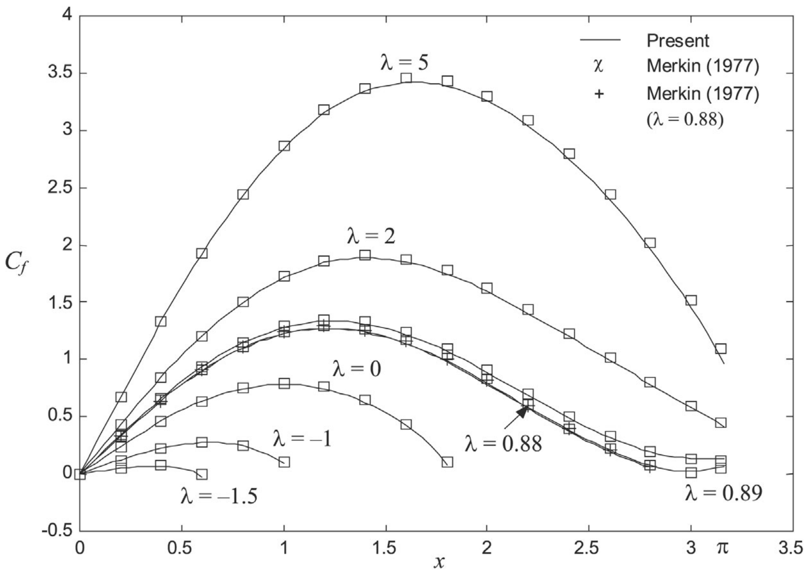

Figure 3. Comparison of the skin friction coefficient C_f for $K=0$ (Newtonian fluid), $Pr = 1$ and various values of λ

and between 1.84 and 1.85 for $K = 0.0$, between 2.14 and 2.15 for $K = 0.5$, and between 2.70 and 2.71 for $K = 2.0$ when $Pr = 6.8$. Figures 4 and 5 also show that for each value of K there is a value of $\lambda = \lambda_0$ (<0) below which a boundary-layer separation is not possible. The reason is that for $\lambda < 0$, the cylinder is cooled and the free convection boundary-layer would start at $x = \pi$

Table III.
Values of heat transfer coefficient Q_w for $K=0$ (Newtonian fluid), $\Pr = 6.8$ and various values of λ

x	λ −2.5	−1.5	−1.0	−0.5	0.0	1.0	1.84	1.85	2.0	5.0	10.0
0.0	0.8898	1.0384	1.0892	1.1324	1.1704	1.2354	1.2822	1.2827	1.2905	1.4205	1.5768
0.2	0.8635	1.0256	1.0782	1.1226	1.1613	1.2274	1.2746	1.2752	1.2830	1.4139	1.5709
0.4		0.9904	1.0483	1.0961	1.1370	1.2059	1.2546	1.2552	1.2632	1.3967	1.5551
0.6		0.9294	0.9984	1.0525	1.0975	1.1712	1.2224	1.2230	1.2314	1.3689	1.5297
0.8			0.9252	0.9907	1.0423	1.1236	1.1784	1.1790	1.1879	1.3312	1.4952
1.0			0.8197	0.9081	0.9709	1.0634	1.1233	1.1239	1.1335	1.2843	1.4520
1.2				0.7982	0.8814	0.9911	1.0578	1.0585	1.0690	1.2289	1.4010
1.4				0.6370	0.7695	0.9069	0.9829	0.9837	0.9953	1.1663	1.3428
1.6					0.6214	0.8112	0.8998	0.9007	0.9137	1.0973	1.2781
1.8					0.3172	0.7037	0.8098	0.8108	0.8256	1.0232	1.2076
2.0						0.5832	0.7145	0.7157	0.7327	0.9451	1.1318
2.2						0.4454	0.6161	0.6175	0.6371	0.8638	1.0506
2.4							0.5170	0.5186	0.5411	0.7795	0.9633
2.6							0.4201	0.4219	0.4473	0.6914	0.8680
2.8							0.3281	0.3300	0.3573	0.5958	0.7592
3.0								0.2535	0.2729	0.4807	0.6218
π								0.2948	0.2448	0.3597	0.4744

Table IV.
Values of skin friction coefficient C_f for $K=0$ (Newtonian fluid), $\Pr = 6.8$ and various values of λ

x	λ −2.5	−1.5	−1.0	−0.5	0.0	1.0	1.84	1.85	2.0	5.0	10.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0226	0.1254	0.1672	0.2058	0.2421	0.3099	0.3630	0.3637	0.3729	0.5439	0.7942
0.4		0.2243	0.3093	0.3872	0.4601	0.5954	0.7012	0.7024	0.7207	1.0598	1.5549
0.6		0.2721	0.4045	0.5234	0.6334	0.8358	0.9932	0.9950	1.0222	1.5237	2.2528
0.8			0.4329	0.5968	0.7453	1.0146	1.2220	1.2244	1.2600	1.9152	2.8622
1.0			0.3749	0.5939	0.7840	1.1203	1.3756	1.3785	1.4222	2.2193	3.3628
1.2				0.5037	0.7431	1.1480	1.4487	1.4521	1.5033	2.4271	3.7400
1.4				0.3074	0.6209	1.0994	1.4425	1.4464	1.5042	2.5359	3.9856
1.6					0.4150	0.9829	1.3648	1.3691	1.4326	2.5492	4.0973
1.8					0.0591	0.8124	1.2290	1.2335	1.3016	2.4750	4.0777
2.0						0.6061	1.0525	1.0573	1.1283	2.3250	3.9330
2.2						0.3847	0.8552	0.8601	0.9320	2.1118	3.6708
2.4							0.6566	0.6614	0.7314	1.8472	3.2979
2.6							0.4728	0.4772	0.5417	1.5384	2.8172
2.8							0.3130	0.3167	0.3706	1.1845	2.2210
3.0								0.1895	0.2193	0.7664	1.4745
π								0.2097	0.1503	0.3867	0.7791

(the upper stagnation point), and for sufficiently small values of λ, there comes a point where the flow of the stream upwards cannot overcome the tendency of the fluid next to the cylinder to move downwards under the action of the buoyancy forces (Merkin, 1977). This is an unstable situation and whether a boundary-layer can exist at all on the cylinder for $\lambda < \lambda_0$ (<0) is still an open

	$K = 0$ (Newtonian fluid), Pr = 1									$K = 0$ (Newtonian fluid), Pr = 6.8			
	Merkin (1977) (numerical)		Present (numerical)		Merkin (1977) (series)		Present (series (34))			Present (numerical)		Present (series (34))	
λ	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	λ	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
									−2.8	−0.0598	0.8087		
−1.9	−0.0998	0.3847	−0.0982	0.3854					−2.7	0.0182	0.8405		
−1.8	0.0195	0.4099	0.0203	0.4105					−2.6	0.0876	0.8669		
−1.6	0.2092	0.4441	0.2103	0.4445					−2.5	0.1513	0.8898		
−1.4	0.3698	0.4691	0.3706	0.4695					−2.0	0.4229	0.9761		
−1.2	0.5146	0.4894	0.5154	0.4897					−1.5	0.6521	1.0384		
−1.0	0.6489	0.5067	0.6497	0.5071					−1.0	0.8596	1.0892		
−0.8	0.7755	0.5219	0.7763	0.5223					−0.8	0.9381	1.1072		
−0.6	0.8963	0.5357	0.8971	0.5360					−0.6	1.0146	1.1243		
−0.4	1.0122	0.5482	1.0131	0.5486					−0.4	1.0892	1.1404		
−0.2	1.1241	0.5597	1.1250	0.5601					−0.2	1.1621	1.1558		
0.0	1.2326	0.5705	1.2336	0.5708					0.0	1.2336	1.1704		
0.2	1.3381	0.5805	1.3392	0.5809					0.2	1.3037	1.1844		
0.4	1.4410	0.5900	1.4421	0.5904					0.4	1.3725	1.1979		
0.6	1.5416	0.5990	1.5428	0.5993					0.6	1.4403	1.2109		
0.8	1.6401	0.6075	1.6414	0.6079					0.8	1.5069	1.2234		
1.0	1.7367	0.6156	1.7380	0.6160	1.848	0.835	1.840	0.804	1.0	1.5726	1.2354	1.621	1.440
1.4	1.9248	0.6308	1.9263	0.6312	2.003	0.786	1.996	0.763	1.4	1.7012	1.2584	1.732	1.395
1.8	2.1071	0.6448	2.1088	0.6453	2.167	0.765	2.161	0.745	1.8	1.8267	1.2801	1.849	1.379
2.2	2.2843	0.6579	2.2862	0.6583	2.332	0.755	2.326	0.739	2.2	1.9492	1.3006	1.967	1.376
2.6	2.4571	0.6702	2.4592	0.6706	2.496	0.751	2.492	0.737	2.6	2.0692	1.3200	2.085	1.380
3.0	2.6259	0.6817	2.6282	0.6822	2.659	0.751	2.655	0.738	3.0	2.1868	1.3385	2.201	1.387
4.0	3.0332	0.7081	3.0360	0.7085	3.056	0.759	3.054	0.748	4.0	2.4719	1.3815	2.484	1.413
5.0	3.4230	0.7315	3.4264	0.7320	3.441	0.771	3.440	0.763	5.0	2.7462	1.4205	2.757	1.441
6.0	3.7984	0.7528	3.8024	0.7532	3.812	0.785	3.812	0.778	6.0	3.0113	1.4563	3.021	1.470
7.0	4.1618	0.7722	4.1664	0.7727	4.173	0.800	4.173	0.793	7.0	3.2687	1.4894	3.276	1.499
8.0	4.5148	0.7902	4.5201	0.7907	4.525	0.814	4.525	0.808	8.0	3.5192	1.5204	3.524	1.526
9.0	4.8488	0.8069	4.8548	0.8075	4.867	0.828	4.868	0.823	9.0	3.7638	1.5494	3.766	1.552
10.0	5.1948	0.8226	5.2016	0.8233	5.202	0.841	5.204	0.837	10.0	4.0030	1.5768	4.002	1.577

Table V. Values of $F''(0)$ and $-\theta'(0)$ for $K = 0$ (Newtonian fluid), $\mathrm{Pr} = 1$ and $\mathrm{Pr} = 6.8$, and various values of λ

question. We can see from Figures 4 and 5 that λ_0 (<0) increases with increase in the value of K and also increases with increase in the value of Pr.

Following Merkin (1977), we can show that the separation of the boundary-layer will not occur for $\lambda > 1$. From equations (10) and (16), and the boundary conditions (equation (14a)), we get, on $y = 0$,

$$\left(1 + \frac{K}{2}\right)\left(\frac{\partial^3 \psi}{\partial y^3}\right)_{y=0} + (\lambda + \cos x)\sin x = 0 \qquad (35)$$

Though $(\partial^2\psi/\partial y^2)_{y=0} = 0$ at $x = x_s$, the streamwise velocity component $\partial\psi/\partial y$ will be positive near $y = 0$ and so $(\partial^3\psi/\partial y^3)_{y=0} \geq 0$ at $x = x_s$. We thus have from equation (35) that $(\lambda + \cos x)\sin x \leq 0$, which cannot hold in the range $0 \leq x \leq \pi$ for $\lambda > 1$.

x	λ −1.9	−1.5	−1.0	−0.5	0.0	0.5	0.9	0.91	1.0	2.0	5.0
0.0	0.4041	0.4521	0.4918	0.5221	0.5471	0.5685	0.5838	0.5842	0.5874	0.6199	0.6933
0.2	0.3882	0.4444	0.4863	0.5176	0.5431	0.5649	0.5803	0.5807	0.5840	0.6168	0.6906
0.4		0.4221	0.4713	0.5054	0.5324	0.5551	0.5711	0.5715	0.5749	0.6086	0.6834
0.6		0.3755	0.4457	0.4853	0.5150	0.5393	0.5563	0.5567	0.5602	0.5953	0.6719
0.8			0.4059	0.4565	0.4907	0.5176	0.5359	0.5364	0.5402	0.5772	0.6562
1.0			0.3382	0.4175	0.4594	0.4901	0.5103	0.5108	0.5149	0.5547	0.6366
1.2				0.3636	0.4202	0.4568	0.4797	0.4802	0.4849	0.5281	0.6136
1.4				0.2719	0.3717	0.4177	0.4444	0.4450	0.4503	0.4979	0.5874
1.6					0.3087	0.3727	0.4049	0.4056	0.4117	0.4647	0.5586
1.8					0.1960	0.3208	0.3615	0.3623	0.3696	0.4293	0.5275
2.0						0.2592	0.3147	0.3157	0.3246	0.3925	0.4946
2.2						0.1720	0.2651	0.2665	0.2777	0.3552	0.4601
2.4							0.2135	0.2153	0.2302	0.3183	0.4242
2.6							0.1609	0.1637	0.1845	0.2829	0.3865
2.8							0.1112	0.1161	0.1458	0.2493	0.3460
3.0								0.0915	0.1247	0.2171	0.2997
π								0.1050	0.1248	0.1931	0.2635

Table VI.
Values of heat transfer coefficient Q_w for $K=0.5$, $Pr = 1$ and various values of λ

x	λ −1.9	−1.5	−1.0	−0.5	0.0	0.5	0.9	0.91	1.0	2.0	5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0071	0.0736	0.1386	0.1959	0.2483	0.2974	0.3347	0.3357	0.3439	0.4312	0.6650
0.4		0.1154	0.2508	0.3670	0.4721	0.5701	0.6445	0.6463	0.6627	0.8363	1.2994
0.6		0.0871	0.3129	0.4917	0.6505	0.7971	0.9081	0.9109	0.9352	1.1928	1.8766
0.8			0.3007	0.5518	0.7665	0.9617	1.1085	1.1121	1.1441	1.4824	2.3738
1.0			0.1769	0.5327	0.8081	1.0524	1.2339	1.2384	1.2778	1.6923	2.7739
1.2				0.4199	0.7691	1.0640	1.2794	1.2846	1.3312	1.8160	3.0655
1.4				0.1723	0.6483	0.9984	1.2468	1.2527	1.3059	1.8540	3.2441
1.6					0.4446	0.8635	1.1445	1.1512	1.2106	1.8131	3.3110
1.8					0.1129	0.6724	0.9872	0.9945	1.0595	1.7057	3.2727
2.0						0.4396	0.7939	0.8018	0.8719	1.5481	3.1394
2.2						0.1652	0.5867	0.5953	0.6697	1.3587	2.9227
2.4							0.3892	0.3983	0.4765	1.1554	2.6337
2.6							0.2245	0.2343	0.3152	0.9525	2.2799
2.8							0.1159	0.1265	0.2067	0.7584	1.8623
3.0								0.1005	0.1660	0.5719	1.3679
π								0.1337	0.1758	0.4340	0.9377

Table VII.
Values of skin friction coefficient C_f for $K=0.5$, $Pr = 1$ and various values of λ

Conclusions

In this paper we have studied the problem of steady mixed convection boundary-layer flow over an isothermal horizontal circular cylinder immersed in a micropolar fluid. We have sought to determine how the mixed convection parameter λ, the material parameter K and the Prandtl number Pr affect

x	λ −3.0	−2.5	−2.0	−1.5	−1.0	0.0	1.0	2.14	2.15	3.0	5.0	10.0
0.0	0.8298	0.9096	0.9592	1.0040	1.0430	1.1105	1.1659	1.2196	1.2200	1.2553	1.3274	1.4673
0.2	0.8024	0.8941	0.9461	0.9927	1.0329	1.1020	1.1582	1.2125	1.2130	1.2486	1.3212	1.4616
0.4		0.8489	0.9097	0.9620	1.0058	1.0792	1.1378	1.1938	1.1942	1.2307	1.3046	1.4465
0.6		0.7529	0.8440	0.9097	0.9607	1.0421	1.1049	1.1637	1.1641	1.2020	1.2780	1.4222
0.8			0.7238	0.8296	0.8954	0.9906	1.0597	1.1226	1.1231	1.1629	1.2418	1.3892
1.0				0.6988	0.8040	0.9239	1.0025	1.0711	1.0716	1.1142	1.1968	1.3480
1.2					0.6666	0.8407	0.9338	1.0101	1.0107	1.0566	1.1438	1.2993
1.4						0.7375	0.8538	0.9404	0.9411	0.9912	1.0837	1.2438
1.6						0.6038	0.7628	0.8634	0.8641	0.9193	1.0177	1.1822
1.8						0.3755	0.6605	0.7804	0.7812	0.8422	0.9469	1.1153
2.0							0.5457	0.6931	0.6940	0.7617	0.8724	1.0435
2.2							0.4145	0.6037	0.6048	0.6796	0.7953	0.9669
2.4								0.5148	0.5160	0.5973	0.7160	0.8850
2.6								0.4292	0.4305	0.5161	0.6338	0.7959
2.8								0.3494	0.3508	0.4354	0.5457	0.6946
3.0									0.2877	0.3500	0.4412	0.5670
π									0.3259	0.2769	0.3336	0.4290

Table VIII. Values of heat transfer coefficient Q_w for $K=0.5$, $Pr = 6.8$ and various values of λ

x	λ −3.0	−2.5	−2.0	−1.5	−1.0	0.0	1.0	2.14	2.15	3.0	5.0	10.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0112	0.0536	0.1069	0.1453	0.1815	0.2500	0.3125	0.3790	0.3795	0.4265	0.5306	0.7662
0.4		0.0822	0.1856	0.2645	0.3378	0.4754	0.6003	0.7325	0.7336	0.8269	1.0333	1.4994
0.6		0.0538	0.2086	0.3340	0.4470	0.6556	0.8425	1.0391	1.0407	1.1789	1.4839	2.1706
0.8			0.1350	0.3303	0.4900	0.7736	1.0222	1.2811	1.2833	1.4644	1.8625	2.7548
1.0				0.2193	0.4495	0.8175	1.1280	1.4465	1.4492	1.6704	2.1542	3.2322
1.2					0.3004	0.7813	1.1550	1.5298	1.5330	1.7908	2.3505	3.5890
1.4						0.6640	1.1052	1.5323	1.5358	1.8256	2.4492	3.8177
1.6						0.4653	0.9873	1.4618	1.4657	1.7818	2.4546	3.9169
1.8						0.1516	0.8158	1.3320	1.3361	1.6713	2.3755	3.8902
2.0							0.6099	1.1606	1.1649	1.5101	2.2244	3.7445
2.2							0.3919	0.9675	0.9718	1.3156	2.0149	3.4884
2.4								0.7721	0.7762	1.1041	1.7596	3.1293
2.6								0.5900	0.5937	0.8876	1.4663	2.6706
2.8								0.4294	0.4324	0.6706	1.1350	2.1053
3.0									0.3063	0.4457	0.7479	1.3992
π									0.3445	0.2671	0.3993	0.7386

Table IX. Values of skin friction coefficient C_f for $K=0.5$, $Pr = 6.8$ and various values of λ

x	λ −2.1	−1.5	−1.0	−0.5	0.0	0.5	0.7	0.94	0.95	1.0	2.0	5.0
0.0	0.3858	0.4347	0.4637	0.4869	0.5067	0.5240	0.5303	0.5376	0.5379	0.5394	0.5664	0.6284
0.2	0.3743	0.4287	0.4590	0.4829	0.5030	0.5206	0.5270	0.5344	0.5347	0.5362	0.5634	0.6258
0.4		0.4120	0.4462	0.4720	0.4932	0.5115	0.5182	0.5259	0.5262	0.5277	0.5557	0.6190
0.6		0.3815	0.4246	0.4540	0.4773	0.4970	0.5041	0.5121	0.5125	0.5141	0.5432	0.6082
0.8			0.3923	0.4286	0.4552	0.4769	0.4846	0.4933	0.4936	0.4954	0.5263	0.5934
1.0			0.3437	0.3945	0.4267	0.4514	0.4600	0.4696	0.4700	0.4719	0.5051	0.5751
1.2				0.3490	0.3913	0.4207	0.4305	0.4413	0.4417	0.4438	0.4801	0.5535
1.4				0.2815	0.3478	0.3846	0.3962	0.4087	0.4092	0.4116	0.4518	0.5290
1.6					0.2926	0.3431	0.3574	0.3722	0.3728	0.3756	0.4208	0.5020
1.8					0.2078	0.2956	0.3142	0.3324	0.3331	0.3364	0.3877	0.4731
2.0						0.2395	0.2662	0.2897	0.2905	0.2947	0.3534	0.4425
2.2						0.1636	0.2123	0.2449	0.2460	0.2513	0.3189	0.4108
2.4							0.1464	0.1992	0.2007	0.2076	0.2852	0.3779
2.6								0.1543	0.1564	0.1661	0.2535	0.3437
2.8								0.1139	0.1172	0.1310	0.2243	0.3072
3.0									0.1035	0.1125	0.1977	0.2655
π									0.2280	0.1547	0.1794	0.2276

Table X.
Values of heat transfer coefficient Q_w for $K=2$, $Pr = 1$ and various values of λ

x	λ −2.1	−1.5	−1.0	−0.5	0.0	0.5	0.7	0.94	0.95	1.0	2.0	5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0262	0.0557	0.1719	0.2264	0.2769	0.3247	0.3432	0.3650	0.3658	0.3703	0.4564	0.6887
0.4		0.0958	0.3157	0.4257	0.5271	0.6225	0.6593	0.7026	0.7043	0.7132	0.8843	1.3444
0.6		0.1050	0.4070	0.5750	0.7278	0.8705	0.9253	0.9898	0.9924	1.0057	1.2595	1.9388
0.8			0.4230	0.6552	0.8609	1.0505	1.1231	1.2082	1.2116	1.2291	1.5623	2.4478
1.0			0.3385	0.6519	0.9139	1.1506	1.2405	1.3454	1.3497	1.3711	1.7792	2.8532
1.2				0.5541	0.8814	1.1661	1.2727	1.3960	1.4017	1.4269	1.9040	3.1441
1.4				0.3417	0.7639	1.0998	1.2230	1.3648	1.3706	1.3994	1.9381	3.3163
1.6					0.5651	0.9621	1.1017	1.2606	1.2670	1.2990	1.8901	3.3729
1.8					0.2691	0.7693	0.9263	1.1010	1.1081	1.1430	1.7748	3.3222
2.0						0.5404	0.7186	0.9088	0.9162	0.9535	1.6116	3.1766
2.2						0.2865	0.5029	0.7090	0.7170	0.7561	1.4213	2.9501
2.4							0.2999	0.5282	0.5364	0.5766	1.2241	2.6553
2.6								0.3902	0.3986	0.4386	1.0358	2.3014
2.8								0.3138	0.3219	0.3598	0.8648	1.8898
3.0									0.3366	0.3510	0.7099	1.4079
π									0.7123	0.4796	0.6014	0.9909

Table XI.
Values of skin friction coefficient C_f for $K=2$, $Pr = 1$ and various values of λ

	λ											
x	−4.0	−3.0	−2.0	−1.5	−1.0	0.0	1.0	2.70	2.71	5.0	10.0	20.0
0.0	0.7496	0.8395	0.9082	0.9349	0.9600	1.0095	1.0515	1.1117	1.1120	1.1787	1.2933	1.4581
0.2	0.7511	0.8262	0.8982	0.9256	0.9515	1.0020	1.0446	1.1055	1.1058	1.1729	1.2881	1.4532
0.4		0.7882	0.8708	0.9006	0.9284	0.9818	1.0262	1.0889	1.0892	1.1577	1.2743	1.4404
0.6		0.7121	0.8241	0.8588	0.8902	0.9490	0.9965	1.0622	1.0626	1.1333	1.2521	1.4198
0.8			0.7524	0.7975	0.8358	0.9035	0.9557	1.0259	1.0263	1.1001	1.2219	1.3916
1.0			0.6327	0.7095	0.7621	0.8449	0.9043	0.9806	0.9810	1.0588	1.1843	1.3564
1.2				0.5603	0.6607	0.7725	0.8425	0.9271	0.9275	1.0102	1.1400	1.3146
1.4					0.4862	0.6841	0.7708	0.8663	0.8668	0.9553	1.0898	1.2667
1.6						0.5733	0.6894	0.7995	0.8000	0.8952	1.0343	1.2132
1.8						0.4114	0.5986	0.7283	0.7288	0.8312	0.9743	1.1545
2.0							0.4975	0.6543	0.6550	0.7644	0.9106	1.0909
2.2							0.3832	0.5798	0.5805	0.6961	0.8433	1.0220
2.4								0.5070	0.5077	0.6270	0.7724	0.9473
2.6								0.4381	0.4388	0.5571	0.6965	0.8647
2.8								0.3760	0.3767	0.4848	0.6120	0.7695
3.0									0.3474	0.4047	0.5085	0.6495
π									0.3857	0.3333	0.4022	0.5260

Table XII. Values of heat transfer coefficient Q_w for $K=2$, $Pr = 6.8$ and various values of λ

	λ											
x	−4.0	−3.0	−2.0	−1.5	−1.0	0.0	1.0	2.70	2.71	5.0	10.0	20.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.0156	0.0424	0.1559	0.1863	0.2166	0.2803	0.3394	0.4325	0.4331	0.5485	0.7788	1.1862
0.4		0.0682	0.2835	0.3451	0.4062	0.5339	0.6518	0.8370	0.8381	1.0670	1.5227	2.3280
0.6		0.0580	0.3574	0.4522	0.5454	0.7383	0.9146	1.1898	1.1913	1.5298	2.2015	3.3860
0.8			0.3521	0.4863	0.6147	0.8753	1.1095	1.4716	1.4736	1.9158	2.7892	4.3254
1.0			0.2274	0.4260	0.5985	0.9327	1.2248	1.6694	1.6719	2.2098	3.2658	5.1170
1.2				0.2256	0.4819	0.9053	1.2557	1.7776	1.7805	2.4035	3.6178	5.7387
1.4					0.2170	0.7939	1.2056	1.7979	1.8012	2.4962	3.8391	6.1758
1.6						0.6034	1.0853	1.7397	1.7432	2.4939	3.9299	6.4203
1.8						0.3250	0.9123	1.6182	1.6219	2.4080	3.8960	6.4705
2.0							0.7090	1.4529	1.4568	2.2536	3.7469	6.3291
2.2							0.5009	1.2649	1.2687	2.0471	3.4934	6.0011
2.4								1.0734	1.0769	1.8026	3.1451	5.4910
2.6								0.8925	0.8957	1.5297	2.7071	4.7985
2.8								0.7314	0.7338	1.2298	2.1746	3.9103
3.0									0.6475	0.8919	1.5186	2.7764
π									0.7191	0.6079	0.9182	1.7285

Table XIII. Values of skin friction coefficient C_f for $K=2$, $Pr = 6.8$ and various values of λ

	$K = 0.5$, Pr = 1				$K = 2.0$, Pr = 1			
	(numerical)		(series (34))		(numerical)		(series (34))	
λ	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$	$F''(0)$	$-\theta'(0)$
−2.4					−0.0505	0.3428		
−2.3					0.0068	0.3618		
−2.2					0.0426	0.3734		
−2.1					0.0869	0.3858		
−2.0	−0.0152	0.3858			0.1276	0.3964		
−1.9	0.0656	0.4041			0.1658	0.4059		
−1.8	0.1370	0.4189			0.1941	0.4130		
−1.6	0.2639	0.4427			0.2625	0.4282		
−1.4	0.3700	0.4608			0.3264	0.4414		
−1.2	0.4753	0.4773			0.3809	0.4532		
−1.0	0.5743	0.4918			0.4448	0.4639		
−0.8	0.6685	0.5047			0.5004	0.4737		
−0.6	0.7588	0.5166			0.5542	0.4829		
−0.4	0.8459	0.5275			0.6064	0.4913		
−0.2	0.9302	0.5376			0.6573	0.4993		
0.0	1.0121	0.5471			0.7069	0.5069		
0.2	1.0920	0.5560			0.7554	0.5141		
0.4	1.1700	0.5645			0.8029	0.5209		
0.6	1.2463	0.5725			0.8495	0.5273		
0.8	1.3212	0.5801			0.8954	0.5336		
1.0	1.3946	0.5874	1.4990	0.7672	0.9404	0.5396	0.8688	0.6573
1.4	1.5379	0.6012	1.6275	0.7270	1.0285	0.5509	0.9566	0.6310
1.8	1.6769	0.6139	1.7524	0.7098	1.1142	0.5615	1.0458	0.6215
2.2	1.8122	0.6258	1.8794	0.7028	1.1978	0.5714	1.1345	0.6191
2.6	1.9442	0.6370	2.0062	0.7011	1.2795	0.5808	1.2218	0.6205
3.0	2.0732	0.6475	2.1320	0.7023	1.3595	0.5897	1.3076	0.6238
4.0	2.3849	0.6717	2.4401	0.7118	1.5530	0.6101	1.5157	0.6360
5.0	2.6834	0.6933	2.7384	0.7249	1.7389	0.6285	1.7153	0.6502
6.0	2.9712	0.7129	3.0274	0.7391	1.9183	0.6452	1.9077	0.6647
7.0	3.2522	0.7310	3.3081	0.7534	2.0923	0.6607	2.0938	0.6788
8.0	3.5262	0.7479	3.5815	0.7674	2.2616	0.6750	2.2745	0.6924
9.0	3.7902	0.7634	3.8482	0.7809	2.4323	0.6889	2.4505	0.7054
10.0	4.0501	0.7781	4.1090	0.7940	2.5937	0.7015	2.6224	0.7178

Table XIV. Values of $F''(0)$ and $-\theta'(0)$ for $K = 0.5$ and 2.0 with Pr = 1

the flow and heat transfer characteristics as well as the position of the boundary-layer separation x_s. Solutions of the governing non-similar equations are obtained numerically using the Keller-box method. From this study we can draw the following conclusions:

- an increase in the value of K leads to a decrease of the heat transfer coefficient Q_w and an increase of the skin friction coefficient C_f;
- an increase in the value of K leads to an increase of the value of $\lambda = \lambda_K$ (>0) which first gives no separation;

λ	$K = 0.5$, Pr = 6.8 (numerical) $F''(0)$	$-\theta'(0)$	(series (34)) $F''(0)$	$-\theta'(0)$	$K = 2.0$, Pr = 6.8 (numerical) $F''(0)$	$-\theta'(0)$	(series (34)) $F''(0)$	$-\theta'(0)$
−4.2					−0.0011	0.7390		
−4.1					0.0291	0.7551		
−4.0					0.0575	0.7696		
−3.5					0.1075	0.7843		
−3.2	−0.0274	0.7835			0.1818	0.8194		
−3.1	0.0281	0.8085			0.2048	0.8297		
−3.0	0.0785	0.8298			0.2272	0.8395		
−2.5	0.2908	0.9096			0.3313	0.8821		
−2.0	0.4486	0.9592			0.4031	0.9082		
−1.5	0.6012	1.0040			0.4789	0.9349		
−1.0	0.7450	1.0430			0.5639	0.9634		
−0.8	0.8021	1.0578			0.5967	0.9739		
−0.6	0.8579	1.0718			0.6289	0.9841		
−0.4	0.9125	1.0853			0.6606	0.9939		
−0.2	0.9660	1.0981			0.6918	1.0033		
0.0	1.0185	1.1105			0.7225	1.0125		
0.2	1.0702	1.1223			0.7528	1.0213		
0.4	1.1210	1.1338			0.7826	1.0298		
0.6	1.1710	1.1448			0.8121	1.0382		
0.8	1.2203	1.1555			0.8412	1.0462		
1.0	1.2690	1.1659	1.3369	1.3326	0.8700	1.0541	0.7857	1.1852
1.4	1.3644	1.1857	1.4334	1.3028	0.9265	1.0693	0.8523	1.1501
1.8	1.4576	1.2044	1.5326	1.2952	0.9820	1.0837	0.9174	1.1394
2.2	1.5488	1.2222	1.6316	1.2971	1.0363	1.0975	0.9805	1.1392
2.6	1.6381	1.2391	1.7293	1.3037	1.0897	1.1108	1.0416	1.1442
3.0	1.7258	1.2553	1.8153	1.3127	1.1422	1.1235	1.1008	1.1520
4.0	1.9387	1.2930	2.0281	1.3399	1.2700	1.1533	1.2568	1.1766
5.0	2.1438	1.3274	2.2310	1.3587	1.3856	1.1787	1.3742	1.2034
6.0	2.3423	1.3591	2.4355	1.3870	1.5132	1.2061	1.5250	1.2301
7.0	2.5351	1.3886	2.6227	1.4142	1.6297	1.2298	1.6501	1.2558
8.0	2.7249	1.4165	2.8136	1.4402	1.7434	1.2522	1.7905	1.2803
9.0	2.9083	1.4424	3.0090	1.4648	1.8546	1.2733	1.9268	1.3037
10.0	3.0901	1.4673	3.1895	1.4883	1.9635	1.2933	2.0595	1.3259

Table XV. Values of $F''(0)$ and $-\theta'(0)$ for $K = 0.5$ and $K = 2.0$ with $\mathrm{Pr} = 6.8$

- an increase in the value of K leads to an increase of the value of $\lambda = \lambda_0$ (<0) below which a boundary-layer solution is not possible;
- an increase in the value of Pr leads to an increase of both the heat transfer coefficient Q_w and the skin friction coefficient C_f;
- an increase in the value of Pr leads to an increase of the value of $\lambda = \lambda_K$ (>0) which first gives no separation;
- an increase in the value of Pr leads to an increase of the value of $\lambda = \lambda_0$ (<0) below which a boundary-layer solution is not possible.

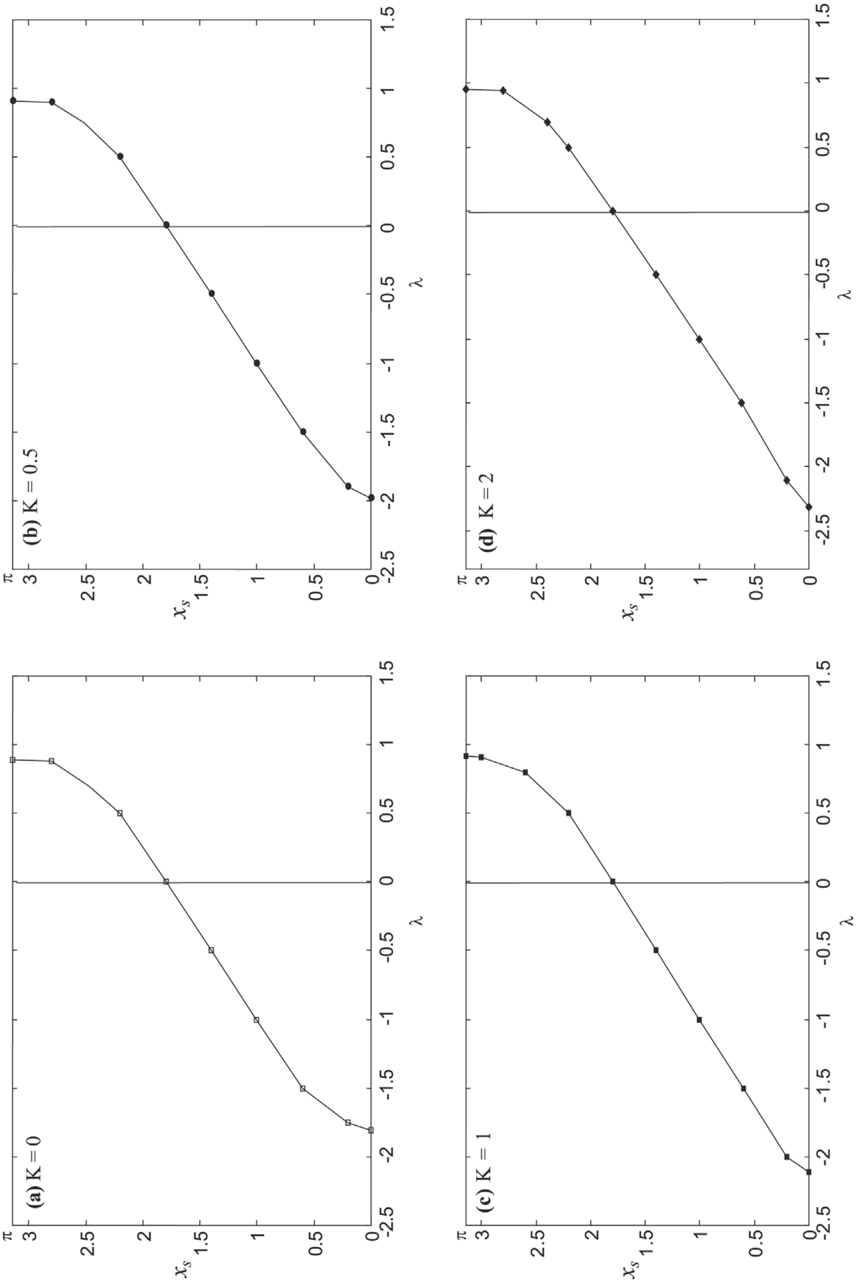

Figure 4.
Variation of the separation point x_s with λ for Pr=1: (a) $K = 0.0$ (Newtonian fluid); (b) $K = 0.5$; (c) $K = 1.0$; (d) $K = 2.0$

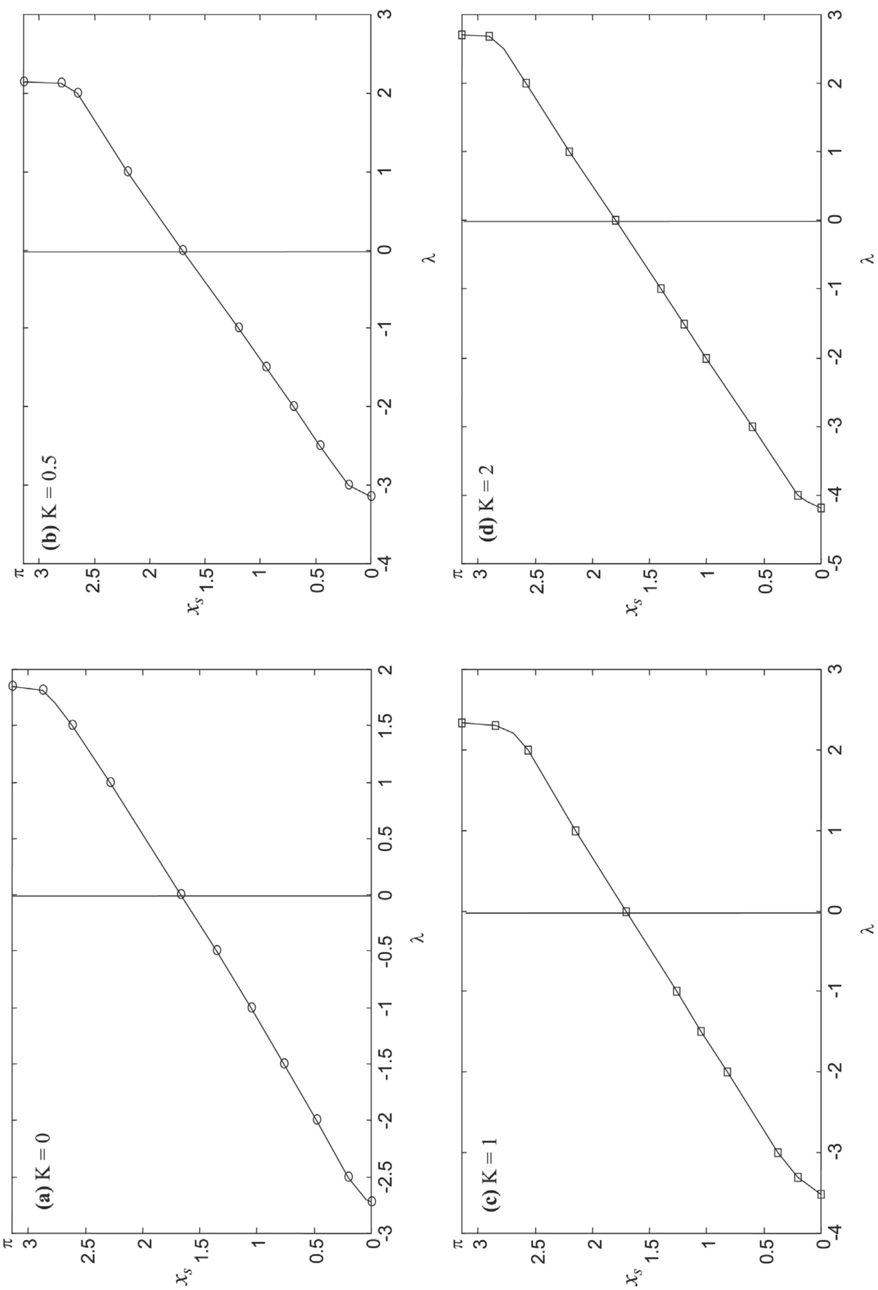

Figure 5.
Variation of the separation point x_s with λ for Pr=6.8: (a) $K = 0.0$ (Newtonian fluid); (b) $K = 0.5$; (c) $K = 1.0$; (d) $K = 2.0$

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